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Relationship between work-hardening exponent and load dependence of Vickers hardness in copper

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Micro-Vickers hardness measurements were conducted on pure copper under cold-worked and annealed conditions at loads ranging 0.147 to 9.8 N as well as tensile tests. Characteristics of the load dependence of the Vickers hardness (H_V) of these specimens were compared with the work-hardening exponents, n, obtained through tensile tests. There was a trend that the slope of the load dependence of the hardness was larger in copper with a smaller n. The slope, S, was a good measure for correlating with n, and n could be expressed as $n = -0.293/S$. The 0.2% offset stress $\sigma_{0.2}$ and ultimate tensile stress σ_{UTS} were estimated by using n determined from the S–n relation and the relations of Cahoon *et al.* and Tabor. The estimated $\sigma_{0.2}$ and σ_{UTS} showed good coincidence with those obtained from tensile tests.

1. Introduction

To date, the Vickers hardness test has been widely used to evaluate mechanical properties of metals because of its simple testing technique. The Vickers hardness test adopts a diamond pyramid as an indenter which makes geometrically similar depressions regardless of their size. The hardness, therefore, represents only an index of a stress at a given strain, that is, the Vickers hardness corresponds to a true stress at a true strain of 8% in tensile tests [1]. This is the reason why it is believed that strain-dependent properties, for example the work-hardening exponent, cannot be obtained by solely using hardness tests with pyramidal indenters, while the strain-hardening properties can be obtained by hardness tests with spherical indenters [1–3].

Taking work-hardening effects of metals into consideration, Tabor [1] presented a relationship among the Vickers hardness (H_V), the work-hardening exponent (n) and the ultimate tensile stress, σ_{UTS}, and Cahoon *et al.* [4] proposed a relationship among H_V, n and the 0.2% offset stress, $\sigma_{0.2}$. The strain-hardening exponent involved in their relations, however, could not be obtained by the Vickers hardness test or hardness tests with pyramidal indenters, but by other testing techniques such as tensile tests, the Meyer hardness test and the Brinell hardness tests, because of the reason mentioned above. Recently, nanoindentation techniques have been applied to evaluate the mechanical properties of ion-irradiated metals [5–7], but main attention was paid to the relative change in hardness rather than to the change in the yield stress and the work-hardening exponent which have clear physical meanings, because of the same reason. The values of $\sigma_{0.2}$ and σ_{UTS} can be evaluated through the Vickers hardness test and nanoindentation techniques together with the relations of Cahoon *et al.* and Tabor, once n can be derived from those hardness tests.

On the other hand, it has been known that the hardness measured with pyramidal indenters depends on load, especially at small loads [8–11]. Recently, Atkinson and Shi [12, 13] suggested that the load dependence of the Vickers hardness reflected work-hardening properties of the metal. Hitherto, however, there has been little definitive and systematic work on the relationship between the load dependence of the Vickers hardness and the work-hardening properties of metals, except for a few works [14, 15].

The purposes of the present study are to examine the relationship between the load dependence of H_V and the strain-hardening exponent, and to estimate $\sigma_{0.2}$ and σ_{UTS} using the relations of Cahoon *et al.* and Tabor through the Vickers hardness test. To this end, the determinations of load dependence of H_V as well as tensile tests were carried out on pure copper under cold-worked and annealed conditions, which had different work-hardening exponents.

2. Experimental procedure

Rods of pure copper with 10 mm diameter were annealed in a vacuum of 1.3×10^{-3} Pa for 3.6 ks at 673 K. The annealed copper rods were cold-swaged to reduction ratios of area of 0, 3.5, 7.8, 10.1 and 13.5%. Hereafter, these cold-worked rods are respectively designated as Cu0, Cu3, Cu8, Cu10 and Cu14 specimens for convenience. In addition to these specimens, a rod which was obtained by annealing the pure copper in a vacuum of 1.3×10^{-3} Pa for 7.2 ks at 573 K was made, and this rod is called the CuA specimen. The names of specimens and the conditions of cold working and heat treatment are summarized in

TABLE I Names of specimens and conditions of heat-treatment and cold-working

Specimen	Heat-treatment and cold-working
CuA	Annealed at 573 K for 7.2 ks
Cu0	Annealed at 673 K for 3.6 ks
Cu3	Cu0 + 3.5% cold-swaged
Cu8	Cu0 + 7.8% cold-swaged
Cu10	Cu0 + 10.1% cold-swaged
Cu14	Cu0 + 13.5% cold-swaged

Table I. Test pieces for tensile and Vickers hardness tests were prepared from each specimen listed in Table I.

Test pieces for tensile tests were obtained by machining each specimen. The gauge length of test pieces was 30 mm and the diameter of the gauge section was 3 mm. Tensile tests were carried out by using an Instron-type testing machine at an initial tensile strain rate of $2.8 \times 10^{-4}\ s^{-1}$ at room temperature. Five tensile tests were carried out for each specimen. The values of $\sigma_{0.2}$, σ_{UTS} and n were determined by averaging over five tests.

Test pieces for the hardness test were prepared by cutting each specimen into discs with 10 mm diameter and about 10 mm thickness using a low-speed cutter and by polishing the disc surfaces with abrasive papers. Finally, the discs for the hardness test were electropolished to eliminate a surface layer of about 200 μm in order to eliminate the effect on the hardness of damage introduced during the mechanical polishing. The Vickers hardness was measured at ten different loads ranging from 0.147 N (15 gf) to 9.8 N (1 kgf) using a micro-Vickers hardness tester and a loading time of 30 s. Hereafter, loads used in the Vickers hardness test will be expressed by using a unit of gf for convenience. Twenty indentations were made at each load, and the hardness was determined by averaging over twenty indentations.

3. Results and discussion

3.1. Relationship between σ_{UTS} and H_V and between $\sigma_{0.2}$ and H_V

Tabor [1] semi-empirically derived Equation 1 below as the relationship among H_V, σ_{UTS} and n from the experimental results that H_V at a large load was almost equal to the stress at a true strain of 8% in tensile or compression tests of many metals. On deriving Equation 1, he adopted assumptions that the plastic behaviour of metals was described by the well-known constitutive equation that $\sigma_t = K\varepsilon_p^n$ and that the onset of plastic instability occurred at a strain where $dF/d\varepsilon_p = 0$, where σ_t is true stress, F load, K the strength coefficient and ε_p true plastic strain:

$$\sigma_{UTS}\,(\mathrm{MPa}) = 3.27 H_V (1-n)\left(\frac{12.5n}{1-n}\right)^n \qquad (1)$$

where H_V is in units of $kg\,mm^{-2}$. Cahoon *et al.* [4] proposed Equation 2 below as the relationship among H_V, $\sigma_{0.2}$ and n for many metals and alloys. They also used the same assumption as that used by Tabor except for the instability criterion:

$$\sigma_{0.2}\,(\mathrm{MPa}) = 3.27 H_V (0.1)^n \qquad (2)$$

where H_V has the same units as for Equation 1. In these relations, however, n had not been obtained by solely using the Vickers hardness test, as already mentioned. The values of $\sigma_{0.2}$ and σ_{UTS} can be obtained by using Equations 1 and 2, once the Vickers hardness test can provide information about the work-hardening exponent. The method for evaluating n from the Vickers hardness test will be described in a later section.

3.2. Tensile tests

The load–elongation curves of the specimens were converted into σ_t versus ε_p curves. Fig. 1 shows examples of the double logarithmic plot of σ_t and ε_p for Cu0 and Cu14 specimens. The curves are non-linear in all specimens, as shown for Cu0 and Cu14 specimens in Fig. 1. In the present work the work-hardening exponent n, the slope of the $\log\sigma_t$–$\log\varepsilon_p$ curve, was determined by fitting the well-known relation $\sigma_t = K\varepsilon_p^n$ with experimental data at strains larger than 1.0% using the least-squares method; Equations 1 and 2 were derived on the basis of the equation $\sigma_t = K\varepsilon_p^n$ and the value of 0.1 in the right-hand side of Equation 2 was determined by fitting this relation with experimental data at strains larger than about 1%. The values of $\sigma_{0.2}$, σ_{UTS} and n and their standard deviation for each specimen, which were determined by tensile tests, are summarized in Table II.

3.3. Load dependence of Vickers hardness

It was found that values of H_V of metals depended on load, especially at small loads [8, 9, 12, 13], and Atkinson and Shi [12, 13] suggested that the load dependence of H_V reflected the strain-hardening propensity. However, they did not systematically investigate the relation of the load dependence of H_V to the work-hardening propensity. In the present study, therefore, the correlation between the load dependence of H_V and n was systematically examined on the specimens listed in Table I, which had different work-hardening exponents. Fig. 2 shows the load dependence of the Vickers hardness of each specimen. To avoid overlapping of the data, the load-dependence curves are separated into Fig. 2a and b. The value of H_V decreases with increasing load in all specimens, and the numbers in the figure show the values of n of those specimens determined by tensile tests. One can see from Fig. 2 that there is a trend that the slope of the H_V–load curve is greater in a specimen with a smaller n. This result suggests that there is a relationship between the load dependence of H_V and the work-hardening exponent n, and it also suggests a possibility for evaluating n through an adequate analysis of H_V–load curves.

In order to analyze these H_V–load curves, the explicit formula describing the relationship between H_V and load (P) is required. Many workers have found the load dependence of H_V, but few definitive formulae

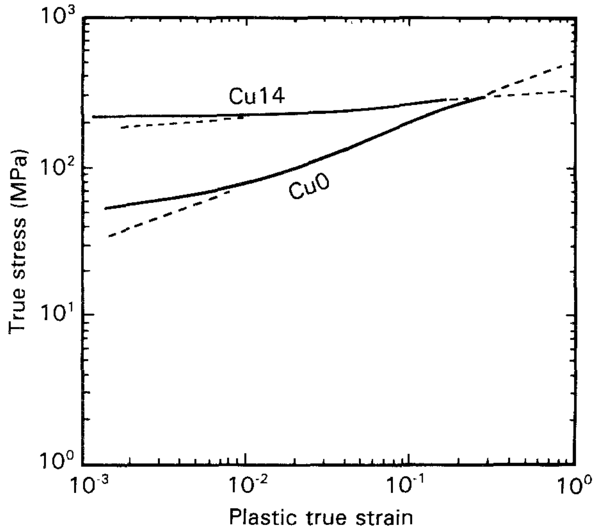

Figure 1 Examples of log–log plots of true stress against true plastic strain for Cu0 and Cu14 specimens. The broken lines show the slope of the curves by fitting $\sigma_t = K\varepsilon_p^n$ to the experimental data at strains larger than 1%.

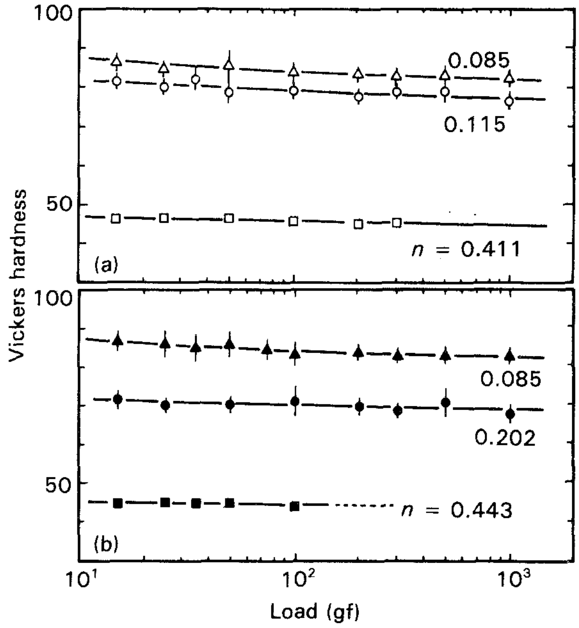

Figure 2(a, b) Load dependence of Vickers hardness in (■) CuA, (□) Cu0, (●) Cu3, (○) Cu8, (▲) Cu10 and (△) Cu14 specimens. The numbers in the figure are the average values of the strain exponents of those specimens. The solid curves are calculated ones obtained by fitting Equation 3 to the experimental data using the least-squares method. 1000 gf = 9.8 N.

TABLE II Values of $\sigma_{0.2}$, σ_{UTS} and n and their standard variation obtained from tensile tests

Specimen	$\sigma_{0.2}$ (MPa)	σ_{UTS} (MPa)	n
CuA	46.6 ± 2.1	217.6 ± 0.4	0.443 ± 0.002
Cu0	58.0 ± 6.5	217.5 ± 2.0	0.411 ± 0.017
Cu3	148.6 ± 4.7	229.4 ± 0.8	0.202 ± 0.019
Cu8	201.7 ± 1.4	233.8 ± 0.2	0.115 ± 0.004
Cu10	219.1 ± 1.9	239.1 ± 0.4	0.085 ± 0.005
Cu14	223.5 ± 2.3	242.6 ± 0.8	0.085 ± 0.004

TABLE III Values of A', λ and B in Equation 3, $A'\lambda$ and $(dH_V/d\log P)_{15}$ for each specimen

Specimen	A'	λ	B	$A'\lambda$	$(dH_V/d\log P)_{15}$
CuA	13.89	0.029	32.57	0.40	− 0.86
Cu0	15.98	0.020	31.45	0.32	− 0.71
Cu3	21.43	0.027	51.59	0.58	− 1.25
Cu8	21.09	0.062	63.70	1.31	− 2.55
Cu10	11.84	0.291	81.35	3.45	− 3.61
Cu14	10.89	0.210	79.72	2.27	− 2.95

describing the relation of H_V to P have yet been obtained. Here, an equation for expressing the relationship between H_V and P is assumed, based on the results in Fig. 2 where H_V decreases exponentially with $\log P$ and asymptotically reaches a constant value at a larger load, as follows:

$$\begin{aligned} H_V &= A\exp(-\lambda \log P) + B \\ &= A'P^{-\lambda} + B \end{aligned} \quad (3)$$

where A, A', B and λ are numerical constants depending on the specimen. The solid curves in Fig. 2 are obtained by fitting Equation 3 to the experimental data of the load dependence of H_V, using the least-squares method. In the Cu0 and CuA specimens, H_V decreased abruptly at larger loads (larger than 500 gf for the Cu0 specimen and 200 gf for the CuA specimen). This may arise from very soft nature of these specimens. Therefore the values of H_V in this load range were not used for calculation. Fig. 2 shows that Equation 2 expresses well the load dependence of H_V. The calculated values of A', λ and B for the specimens are summarized in Table III.

3.4. Relationship between n and load dependence of H_V

As mentioned in the previous section, there is a trend that the slope of H_V–P curves correlates with the strain-hardening exponent n. Hence, the slope of H_V–P curves at $P = 15$ gf, $(dH_V/d\log P)_{15}$, which is the smallest load used in the present measurements, is tentatively chosen as a parameter relating to n in the present work. The values of $-(dH_V/d\log P)_{15}$ for the specimens are shown as a function of n obtained from tensile tests in Fig. 3 and are summarized in Table III. The value of $-(dH_V/d\log P)_{15}$ decreases with increasing n and reaches zero asymptotically at larger n, although there is a scatter of experimental data. In the present work, the following relationship is assumed for the first approximation of the relationship between n and the slope of the H_V–P curve on the basis of the results shown in Fig. 3, because the physical meaning of the relationship between the load dependence of H_V and n is not known at present, usually expressed by saying that the hardness of a material is a poorly defined term and that the hardness is only a measure of the resistance to plastic deformation for metals [16]:

$$n = C \Big/ \left(\frac{dH_V}{d\log_P}\right)_{15} \quad (4)$$

where C is a numerical constant independent of the specimen. The value of C was estimated to be − 0.293

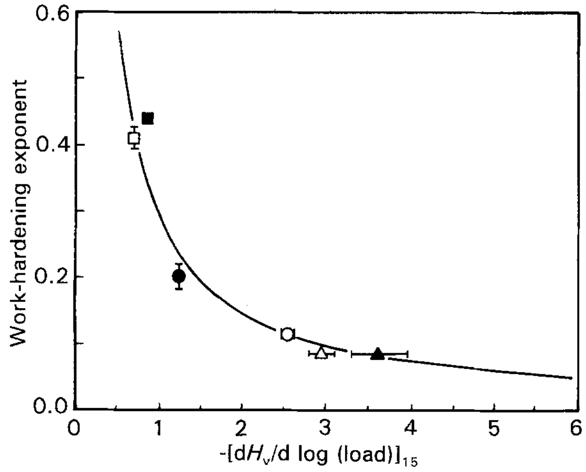

Figure 3 Correlation between n obtained from tensile tests and the slope of the H_V–load curve at a load of 15 gf, $-[dH_V/d\log(\text{load})]_{15}$. The solid curve is determined by the least-squares method for Equation 4. (■) CuA, (□) Cu0, (●) Cu3, (○) Cu8, (▲) Cu10, (△) Cu14.

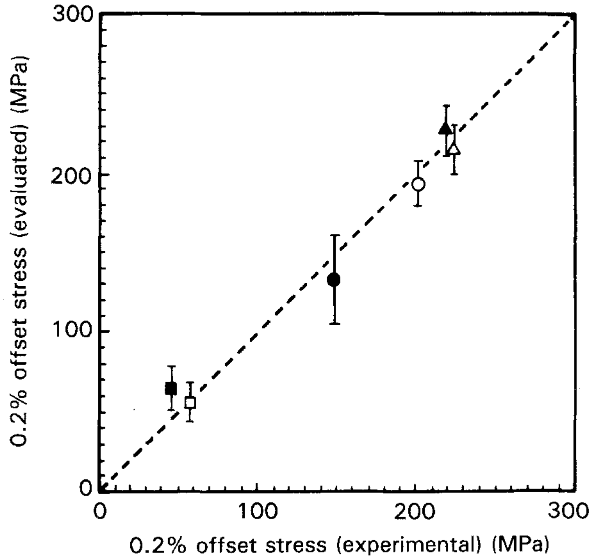

Figure 4 Relationship between 0.2% offset stress obtained by tensile tests and that obtained from Equation 1. The broken line indicates a correspondence between the values of both the 0.2% offset stresses. (■) CuA, (□) Cu0, (●) Cu3, (○) Cu8, (▲) Cu10, (△) Cu14.

$\pm$ 0.023 using the least-squares method. The solid curve in Fig. 3 is obtained by fitting Equation 4 with the experimental data. Equation 4 can describe the relationship between n and the slope of the H_V–P curve. The value of n can be obtained from Equation 4 through measurements of the load dependence of H_V of a metal, once Equation 4 is established for such a metal.

The parameter $(dH_V/d\log P)_{15}$ may not always be a good one relating to n, because it is a derivative value of the H_V–P curve and contains a relatively large experimental error. An alternative parameter is probably $A'\lambda$, and it is possible to obtain an equation similar to Equation 4 for the relation between $A'\lambda$ and n. The value of $A'\lambda$ for each specimen is also listed in Table III.

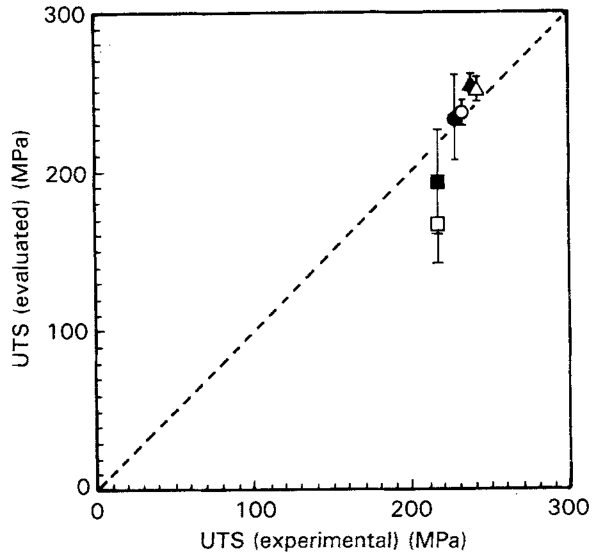

Figure 5 Relationship between the ultimate tensile stress obtained by tensile tests and that through Equation 2. The broken line shows the former value equal to the latter. (■) CuA, (□) Cu0, (●) Cu3, (○) Cu8, (▲) Cu10, (△) Cu14.

3.5. Evaluation of $\sigma_{0.2}$ and σ_{UTS} from the hardness test

The value of n can be evaluated through the load dependence of H_V using Equation 4 as mentioned in the previous section. Therefore the values of $\sigma_{0.2}$ and σ_{UTS} can also be evaluated by using the evaluated n together with Equation 1 and 2. Fig. 4 shows the correlation between $\sigma_{0.2}$ obtained from Equation 1 and that obtained from tensile tests. In the calculation of $\sigma_{0.2}$ using Equation 1, the value of H_V at a load of 200 gf was used. The broken line in the figure indicates the former value equal to the latter. It can be easily seen from Fig. 4 that the evaluated $\sigma_{0.2}$ agrees well with the experimental value within an error of $\pm$ 10%. The evaluated σ_{UTS} is plotted against the experimental value in Fig. 5. The value of H_V at a load of 200 gf was also used in the calculation of σ_{UTS}. The broken line in the figure indicates the evaluated value equal to the experimental one. It is found from Fig. 5 that the evaluated σ_{UTS} agrees with the experimental value, though the estimated value is fairly different from the experimental one for the Cu0 specimen.

4. Conclusions

Based on the present experimental results and discussion, the following conclusions are drawn:

1. The value of H_V depends on load. This load dependence is expressed as $H_V = A'P^{-\lambda} + B$.
2. The slope of the curve of the load dependence of H_V at a load of 15 gf, $(dH_V/d\log P)_{15}$, and $A'\lambda$ are good measures for evaluating the strain hardening exponent n. The value of n can be expressed approximately as $n = -(0.293 \pm 0.023)/(dH_V/d\log P)_{15}$.
3. The 0.2% offset stress and the ultimate tensile stress can be evaluated from the equation $\sigma_{0.2}$ (MPa) $= 3.27H_V(0.1)^n$ and σ_{UTS} (MPa) $= 3.27H_V(1-n)[12.5n/(1-n)]^n$ together with the evaluated value of n from Equation 4.

4. The method presented in the present work for estimating n through the Vickers hardness test together with the relations of Tabor and Cahoon *et al.* can be applicable to the evaluation of n, $\sigma_{0.2}$ and σ_{UTS} of metals through indentation tests with other pyramidal indenters.

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